

OCR

Oxford Cambridge and RSA

Monday 14 May 2018 – Afternoon

AS GCE MATHEMATICS

4725/01 Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

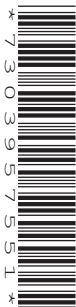
OCR supplied materials:

- Printed Answer Book 4725/01
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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Answer **all** the questions.

- 1 The matrices **A**, **B** and **C** are given by $\mathbf{A} = \begin{pmatrix} 5a \\ 2 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 7b \\ -3 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 3 & 6 \end{pmatrix}$.

Find

- (i) $5\mathbf{A} - 4\mathbf{B}$, [2]
 (ii) \mathbf{BC} . [2]

- 2 The complex number w has modulus 6 and argument $\frac{2\pi}{3}$. Find $\frac{\sqrt{3} + 2i}{w}$, giving your answer in the form $x + iy$, where x and y are exact real numbers. [5]

- 3 The matrix **D** is given by $\mathbf{D} = \begin{pmatrix} d & 0 \\ 0 & 1 \end{pmatrix}$, where $d \neq 0$.

- (i) Find \mathbf{D}^{-1} . [2]

Matrix **D** represents the transformation P.

- (ii) Describe fully the transformation P. [2]

The transformation T is represented by the matrix $\begin{pmatrix} 0 & 1 \\ -d & 0 \end{pmatrix}$ and is equivalent to the transformation P followed by the transformation Q.

- (iii) Find the matrix that represents the transformation Q and describe fully the transformation Q. [4]

- 4 The loci L_1 , L_2 and L_3 are given by $|z - 3 - 4i| = 2$, $\arg(z - 3 - 4i) = \frac{\pi}{3}$ and $|z| = |z - 12|$ respectively.

- (i) Sketch on a single Argand diagram the loci L_1 , L_2 and L_3 . [6]

- (ii) Indicate, by shading, the region of the Argand diagram for which

$$|z - 3 - 4i| \geq 2, 0 \leq \arg(z - 3 - 4i) \leq \frac{\pi}{3} \text{ and } |z| \leq |z - 12|. \quad [3]$$

- 5 The cubic equation $x^3 + 2x^2 + 3x + 4 = 0$ has roots α , β and γ .

- (i) Use the substitution $x = \frac{1}{u+1}$ to obtain a cubic equation in u with integer coefficients. [4]

- (ii) Hence, or otherwise, find the value of $\left(\frac{1}{\alpha} - 1\right)\left(\frac{1}{\beta} - 1\right)\left(\frac{1}{\gamma} - 1\right)$. [3]

- 6 (i) Find $\sum_{r=1}^n r(r^2 + r - 7)$, giving your answer in a fully factorised form. [5]

A sequence u_0, u_1, u_2, \dots is defined by

$$u_0 = 5, u_n = u_{n-1} + n^3 + n^2 - 7n \text{ for } n \geq 1.$$

- (ii) By considering $\sum_{r=1}^n (u_r - u_{r-1})$, find a formula for u_n in terms of n . [3]

[You do not need to factorise your answer.]

- 7 The complex number $a + 3i$ is a root of the quadratic equation

$$z^2 - (7 + i)z + 16 + ki = 0,$$

where a and k are positive real numbers.

- (i) Find the value of a and the value of k . [7]

- (ii) Hence find the other root of the quadratic equation. [2]

- 8 The matrix \mathbf{A} is given by $\mathbf{A} = \begin{pmatrix} a & 1 & -2 \\ -1 & a & 0 \\ 2a & 3 & 1 \end{pmatrix}$, where a is a real constant.

- (i) Show that \mathbf{A} is non-singular. [4]

- (ii) Find \mathbf{A}^{-1} . [4]

- (iii) Hence solve the three simultaneous equations given below.

$$\begin{aligned} ax + y - 2z &= 2 \\ -x + ay &= 1 \\ 2ax + 3y + z &= 0 \end{aligned}$$

[3]

- (iv) Explain briefly why these equations have a unique solution. [1]

- 9 The matrix \mathbf{M} is given by $\mathbf{M} = \begin{pmatrix} m & m \\ 0 & 1 \end{pmatrix}$, where m is a positive constant.

- (i) Find \mathbf{M}^2 and \mathbf{M}^3 in terms of m . [4]

- (ii) Hence suggest a suitable form for the matrix \mathbf{M}^n , where n is a positive integer, $n \geq 2$. [2]

- (iii) Use induction to prove that your answer to part (ii) is correct. [4]

END OF QUESTION PAPER

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