OCR
Oxford Cambridge and RSA

Monday 14 May 2018 – Afternoon

AS GCE MATHEMATICS

4725/01 Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4725/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Answer **all** the questions.

1 The matrices **A**, **B** and **C** are given by $\mathbf{A} = \begin{pmatrix} 5a \\ 2 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 7b \\ -3 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 3 & 6 \end{pmatrix}$.

Find

(i) $5\mathbf{A} - 4\mathbf{B}$, **[2]**

(ii) $\mathbf{BC}$. **[2]**

2 The complex number w has modulus 6 and argument $\frac{2\pi}{3}$. Find $\frac{\sqrt{3}+2\mathrm{i}}{w}$, giving your answer in the form $x+\mathrm{i}y$, where x and y are exact real numbers. **[5]**

3 The matrix **D** is given by $\mathbf{D} = \begin{pmatrix} d & 0 \\ 0 & 1 \end{pmatrix}$, where $d \neq 0$.

(i) Find $\mathbf{D}^{-1}$. **[2]**

Matrix **D** represents the transformation P.

(ii) Describe fully the transformation P. **[2]**

The transformation T is represented by the matrix $\begin{pmatrix} 0 & 1 \\ -d & 0 \end{pmatrix}$ and is equivalent to the transformation P followed by the transformation Q.

(iii) Find the matrix that represents the transformation Q and describe fully the transformation Q. **[4]**

4 The loci L_1, L_2 and L_3 are given by $|z-3-4\mathrm{i}| = 2$, $\arg(z-3-4\mathrm{i}) = \frac{\pi}{3}$ and $|z| = |z-12|$ respectively.

(i) Sketch on a single Argand diagram the loci L_1, L_2 and L_3. **[6]**

(ii) Indicate, by shading, the region of the Argand diagram for which

$$|z-3-4\mathrm{i}| \geqslant 2,\ 0 \leqslant \arg(z-3-4\mathrm{i}) \leqslant \frac{\pi}{3} \text{ and } |z| \leqslant |z-12|.$$ **[3]**

5 The cubic equation $x^3 + 2x^2 + 3x + 4 = 0$ has roots α, β and γ.

(i) Use the substitution $x = \frac{1}{u+1}$ to obtain a cubic equation in u with integer coefficients. **[4]**

(ii) Hence, or otherwise, find the value of $\left(\frac{1}{\alpha}-1\right)\left(\frac{1}{\beta}-1\right)\left(\frac{1}{\gamma}-1\right)$. **[3]**

6 **(i)** Find $\sum_{r=1}^{n} r(r^2+r-7)$, giving your answer in a fully factorised form. **[5]**

A sequence $u_0, u_1, u_2, \ldots$ is defined by

$$u_0 = 5,\ u_n = u_{n-1} + n^3 + n^2 - 7n \text{ for } n \geqslant 1.$$

(ii) By considering $\sum_{r=1}^{n}(u_r - u_{r-1})$, find a formula for u_n in terms of n. **[3]**

[You do not need to factorise your answer.]

7 The complex number $a + 3\mathrm{i}$ is a root of the quadratic equation

$$z^2 - (7+\mathrm{i})z + 16 + k\mathrm{i} = 0,$$

where a and k are positive real numbers.

(i) Find the value of a and the value of k. **[7]**

(ii) Hence find the other root of the quadratic equation. **[2]**

8 The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} a & 1 & -2 \\ -1 & a & 0 \\ 2a & 3 & 1 \end{pmatrix}$, where a is a real constant.

(i) Show that $\mathbf{A}$ is non-singular. **[4]**

(ii) Find $\mathbf{A}^{-1}$. **[4]**

(iii) Hence solve the three simultaneous equations given below.

$$\begin{aligned} ax + y - 2z &= 2 \\ -x + ay &= 1 \\ 2ax + 3y + z &= 0 \end{aligned}$$

[3]

(iv) Explain briefly why these equations have a unique solution. **[1]**

9 The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} m & m \\ 0 & 1 \end{pmatrix}$, where m is a positive constant.

(i) Find $\mathbf{M}^2$ and $\mathbf{M}^3$ in terms of m. **[4]**

(ii) Hence suggest a suitable form for the matrix $\mathbf{M}^n$, where n is a positive integer, $n \geqslant 2$. **[2]**

(iii) Use induction to prove that your answer to part **(ii)** is correct. **[4]**

END OF QUESTION PAPER